INTRODUCTION

Hydrodynamic Machines

A hydromachine is a device used either for extracting energy from a fluid or to add energy to a fluid. There are many types of hydromachines and Figure 1 below illustrates some common types.

![Hydrodynamic Machines Diagram]

Hydrodynamic machines may be classified according to the direction of energy transfer (energy added or extracted) or the type of action (rotodynamic or positive displacement machines). Rotodynamic machines (momentum transfer machines) have a rotating part (runner, impeller or rotor) that is able to rotate continuously and freely in the fluid. This motion allows an uninterrupted flow of fluid which promotes a steadier discharge than positive displacement machines. Positive displacement machines have a moving boundary (such as a piston or a diaphragm) whereby fluid is drawn or forced into a finite space. This motion causes an intermittent or fluctuating flow and the flow rate is governed by the magnitude of the finite space of the machine and the frequency with which it is filled and emptied. The term "pump" is used when the fluid is a liquid,
however, when the fluid is a gas, terms such as compressors, or fans (or blowers) are used. A compressor is a machine whose primary objective is to increase the pressure of the gas, which is accompanied by an increase in the density of the gas. A fan or blower is a machine whose primary objective is to move the gas. Static pressures remain almost unchanged, and therefore the density of the gas is also not changed.

**Basic Equations for Rotodynamic Machinery**

The flow patterns in rotodynamic machines are three-dimensional, with significant viscous effects and flow separation patterns. However, an analytical analysis requires simplification of the actual conditions. Therefore, viscosity is neglected and an idealized two-dimensional flow is assumed throughout the rotor region. Figure 2 below defines the control volume that encompasses the impeller region, which contains a series of vanes. In the control volume, the flow enters through the inlet control surface and exits through the outlet surface.

![Figure 2](image)

A portion of the control volume, at an instant in time, is shown in Figure 3 below. The idealized velocity vectors are diagrammed at the inlet, location 1, and the outlet, location 2. The parameters used in the diagram are defined below.

- \( v \) = absolute velocity of fluid
- \( u \) = peripheral (or circumferential) velocity of blade (\( u = r\omega \))
- \( R \) = fluid velocity relative to blade
- \( v_w \) = velocity of whirl, i.e., component of absolute velocity of fluid in direction tangential to the runner circumference
- \( v_n \) = component of absolute velocity of fluid in a direction normal to the runner circumference
r  =  radius from axis of runner  
ω  =  angular velocity of runner  
Suffix 1 refers to conditions at inlet to runner  
Suffix 2 refers to conditions at outlet from runner.

Figure 3

The power passed on to the rotor from the fluid (or the power passed to the fluid from the rotor) is due to the change in tangential momentum. There may be changes of momentum in other directions also, but the corresponding forces have no moments about the axis of rotation of the rotor.

Now, the torque about a given fixed axis is equal to the rate of increase of angular momentum about the axis. Therefore, the torque on the fluid must be equal to the angular momentum of the fluid leaving the rotor per unit time minus the angular momentum of the fluid entering the rotor per unit time.

The angular momentum of fluid entering the control volume per second, \( I_1 \), is the product of the mass flow rate (\( \dot{m} \)), the tangential velocity (\( v_{w1} \)) and the radius (\( r_1 \)).  
\[
I_1 = \dot{m}v_{w1}r_1 
\]
The angular momentum of fluid leaving the control volume per second, \( I_2 \), is the product of the mass flow rate (\( \dot{m} \)), the tangential velocity (\( v_{w2} \)) and the radius (\( r_2 \)).

\[
I_2 = \dot{m}v_{w2}r_2
\]

However, the mass flow rate, \( \dot{m} \), is the product of the fluid density, \( \rho \), and the flow rate, \( Q \).

\[
\dot{m} = \rho Q
\]

And the torque, \( T \), is given by

\[
T = \rho Q(v_{w2}r_2 - v_{w1}r_1)
\]

The power, \( P \), is the product of torque, \( T \), and the angular velocity, \( \omega \). Therefore,

\[
P = T \omega = \rho Q \omega(v_{w2}r_2 - v_{w1}r_1) = \rho Q(v_{w2}r_2 \omega - v_{w1}r_1 \omega)
\]

However, \( r_1 \omega = u_1 \) and \( r_2 \omega = u_2 \). Therefore,

\[
P = \rho Q(v_{w2}u_2 - v_{w1}u_1)
\]

Now, \( v_{w1} = v_1 \cos \alpha_1 \) and \( v_{w2} = v_2 \cos \alpha_2 \)

Therefore, \( P = \rho Q(v_2u_2 \cos \alpha_2 - v_1u_1 \cos \alpha_1) \)

The power transferred between the fluid and the rotor is given by \( \rho QgH \), where \( H \) is the pressure head across the rotodynamic machine.

For an idealized situation, where there are no losses

\[
\rho QgH = \rho Q(v_2u_2 \cos \alpha_2 - v_1u_1 \cos \alpha_1)
\]

And \( H = \frac{(v_2u_2 \cos \alpha_2 - v_1u_1 \cos \alpha_1)}{g} \), where \( H \) is the theoretical pressure head across the machine.

In the case of a turbine, \( H \) will be negative (as energy is extracted from the fluid) and it is common to use the reverse order of terms in the brackets, for the above equation. For turbines,

\[
H = \frac{(v_1u_1 \cos \alpha_1 - v_2u_2 \cos \alpha_2)}{g}
\]

**Basic Equations for Centrifugal Pumps**

However, for pumps, \( H \) retains the form, \( H = \frac{(v_2u_2 \cos \alpha_2 - v_1u_1 \cos \alpha_1)}{g} \) and can be applied to centrifugal pumps.

If the whirl or tangential component of the velocity is zero at the inlet (\( \alpha_1 = 90^\circ \)), then the flow is completely radial and the maximum head is obtained.
Since \( u_1 v_1 \cos \alpha_1 = 0 \), \( H_{\text{max}} = \frac{(v_2 u_2 \cos \alpha_2)}{g} \)

Now, \( x_2 = \frac{v_{n2}}{\tan \beta_2} = v_{n2} \cot \beta_2 \) and \( v_{w2} = u_2 - x_2 \)

Therefore, \( v_{w2} = u_2 - v_{n2} \cot \beta_2 \) and \( v_2 \cos \alpha_2 = u_2 - v_{n2} \cot \beta_2 \)

\[
H_{\text{max}} = \frac{u_2 (u_2 - v_{n2} \cot \beta_2)}{g} = \frac{(u_2^2 - u_2 v_{n2} \cot \beta_2)}{g} = \frac{u_2^2}{g} - u_2 \frac{v_{n2}}{g} \cot \beta_2
\]

By continuity, \( Q = A_2 v_{n2} = 2\pi r_2 b_2 v_{n2}, \) where \( A_2 \) is the area perpendicular to \( v_{n2}. \)

This implies \( H_{\text{max}} = \frac{u_2^2}{g} - u_2 \frac{Q}{2\pi r_2 b_2} \cot \beta_2 \)

However, \( u_2 = r_2 \omega \) and \( H_{\text{max}} = \frac{r_2^2 \omega^2}{g} - \frac{r_2 \omega}{g} \frac{Q}{2\pi r_2 b_2} \cot \beta_2 = \frac{r_2^2 \omega^2}{g} - \frac{\omega}{g} \frac{Q}{2\pi r_2 b_2} \cot \beta_2 \)

And generally, one can write \( H_{\text{max}} = a_0 - a_1 Q, \) where \( a_0 \) and \( a_1 \) are constants for a given rotodynamic pump. This theoretical head is linearly related to the discharge through the pump; although the y-intercept, \( a_0, \) is constant, the slope of the line (\( a_1 \)) is dependent on the blade angle, \( \beta_2. \) Figure 4 shows the effect of the blade angle. A forward-curving blade have \( \beta_2 > 90^\circ, \) radial blades have \( \beta_2 = 90^\circ \) whilst backward-curving blades have \( \beta_2 < 90^\circ. \)

\[\text{Figure 4 Ideal Characteristic}\]

For real fluids, the theoretical head curve cannot be achieved in practice due to energy losses. The actual output power delivered can be determined using the equation \( \rho Q g H_p \)
where \( H_p \) is the actual head across the pump. If the overall pump efficiency, \( \eta \), is defined as the ratio of the output power to the input power, then the input power is given as

\[
\text{Input Power} = \frac{\rho Q g H_p}{\eta}
\]

In addition, losses affect the H-Q curve, as shown in Figure 5.

**Figure 5 Typical Pump Characteristic**

### Similarity Laws

The development and utilization of hydrodynamic machinery in engineering practice has benefited greatly from the application of dimensional analysis. It has enabled turbine and pump manufacturers to test and develop a relatively small number of turbo machines, and subsequently produce a series of commercial units that cover a broad range of head and flow demands.

Geometric similarity is a pre-requisite of dynamic similarity. For fluid machines the geometric similarity must apply to all significant parts of the system—the rotor, the entrance and discharge passages for pumps. Machines that are geometrically similar in these respects form a **homologous series**.

To obtain the dimensionless \( \Pi \)s, for geometrically similar machines the following variables may be considered:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dimensional formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D )</td>
<td>rotor diameter, here chosen as a suitable measure of the size of the machine</td>
</tr>
<tr>
<td>( Q )</td>
<td>volume rate of flow through the machine</td>
</tr>
<tr>
<td>( N )</td>
<td>rotational speed</td>
</tr>
<tr>
<td>( H )</td>
<td>difference of head across machine, i.e., energy per unit weight</td>
</tr>
<tr>
<td>( g )</td>
<td>Weight per unit mass</td>
</tr>
</tbody>
</table>
However, since $g$ only enters the analysis as a variable associated with $H$, the term $gH$ is used, rather than $g$ or $H$ separately, to simplify the analysis. Gravity does not enter the problem in any other way.

Using the Tabular method, the following dimensionless $\Pi$s can be derived for geometrically similar machines.

\[
\begin{align*}
\Pi_1 &= \frac{Q}{ND^3} \\
\Pi_2 &= \frac{gH}{N^2D^2} \\
\Pi_3 &= \frac{\mu}{\rho ND^2} \quad \text{and} \quad \Pi_4 = \frac{P}{\rho N^3D^3}
\end{align*}
\]

Alternatively $\Pi_3$ may be re-written as $\Pi_3 = \frac{\rho ND^2}{\mu}$, which resembles the Reynolds number.

### Similarity Laws applied to Centrifugal Pumps

These similarity relationships may be used to obtain the optimum performance when a given pump is used in a different location or the performance of two different pumps.

These equations are used and based on the fact that most centrifugal pumps either:

1. Have a variable speed motor, so that the pump can be changed to obtain the required head-discharge relationship while retaining the same impeller (constant $D$), or
2. Have a constant speed motor, so that the pump speed is fixed (constant $N$) and consequently different diameter impellers have to be used to vary the required head-discharge relationship.

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pi_1$</td>
<td>$\frac{Q_A}{N_A} = \frac{Q_B}{N_B}$</td>
</tr>
<tr>
<td>$\Pi_2$</td>
<td>$\frac{H_A}{N_A^2} = \frac{H_B}{N_B^2}$</td>
</tr>
<tr>
<td>$\Pi_4$</td>
<td>$\frac{P_A}{N_A^2} = \frac{P_B}{N_B^2}$</td>
</tr>
</tbody>
</table>

The relationships are known as the “Affinity Laws”.

Q1.
A pump is fitted with a variable speed motor. At 1200rpm it delivers 0.12m³/s of water. What speed of rotation would be required to increase the discharge to 0.15m³/s?

\[
\frac{Q_A}{N_A} = \frac{Q_g}{N_g}
\]

\[
N_g = N_A \cdot \frac{Q_g}{Q_A} = 1200 \times \frac{0.15}{0.12} = 1500 \text{ rpm}
\]

**Specific speed**

The specific speed of a pump may serve as a guide for pump selection. The dimensionless Πs may be used, and the dependence on the rotor diameter, \(D\), is eliminated.

\[
\frac{Q}{ND^3} = c_1
\]

\[
D^3 = \frac{Q}{NC_1}
\]

\[
D = \left(\frac{Q}{NC_2}\right)^{1/3}
\]

\[
\Rightarrow \left(\frac{Q}{NC_1}\right)^{1/2} = \left(\frac{gH}{N^2c_2}\right)^{1/2}
\]

\[
\frac{Q^{1/3}}{N^{1/8}C_1^{1/2}} = \frac{g^{1/2}H^{1/2}}{N^{1/8}C_2^{1/2}}
\]

\[
\frac{NQ^{1/3}}{N^{1/8}g^{1/2}H^{1/2}} = \frac{C_1^{1/6}}{C_2^{1/2}} = C_3
\]

\[
\frac{N^{2/3}Q^{4/3}}{g^{1/2}H^{1/2}} = C_3
\]

\[
\left(\frac{N^{2/3}Q^{2/3}}{g^{1/2}H^{1/2}}\right)^{3/2} = \nu_3^{3/2} = C_4
\]

\[
\frac{NQ^{1/2}}{(gH)^{3/4}} = C_4 = N_2
\]

\[
N_2 = \frac{NQ^{1/2}}{g^{3/4}}
\]

\(c_1, c_2, c_3\) and \(c_4\) are constants

Where the dimensionless parameter, \(N_s\), is considered to be the Specific Speed. However since the acceleration due to gravity, \(g\), is constant, the specific speed is sometimes given as the dimensional parameter, \(N_s\).
\( N_s' \) is defined as the speed (rpm) at which the pump operating discharges 1\( \text{m}^3/\text{s} \) against 1m head (that is, \( N = N_s \) when \( Q = 1\text{m}^3/\text{s} \) and \( H = 1\text{m} \)).

The specific speed is calculated at the normal operating point of the pump (which may or may not be at the maximum efficiency), with all other variable having the corresponding values. The specific speed is the same for all similar pumps (pumps in a homologous series), regardless of size.

### Performance Characteristics of Centrifugal Pumps

Pumps usually run at constant speed, and of particular interest are:
- The variation in head, \( H \), with the discharge, \( Q \)
- The variation of efficiency, \( \eta \), with the discharge, \( Q \)
- The variation of Power, \( P \), with the discharge, \( Q \)

These relationships are usually supplied by the pump manufacturer, and are provided as pump characteristic curves. Typical characteristic curves are shown in the figure below.

![Figure 6 Typical Pump Characteristic Curves](image)

A particular machine may be tested at a fixed head while the load (and speed \( N \)) is varied, and for these results to be applicable to other pumps in the same homologous series, the characteristic curves is plotted using the dimensionless parameters:

\[
\frac{Q}{N D^2}, \quad \frac{P}{\mu N^3 D^4}, \quad \frac{gH}{N^2 D^2} \quad \text{and} \quad \eta
\]

instead of \( Q, P, H \) and \( \eta \) respectively.
Selection of Centrifugal Pumps

Factors that influence the selection of a pump for a given application are:

- The head-discharge relationship (comparison of various H-Q curves)
- Efficiency
- Power requirements
- Stability of the head-discharge relationships
- Operational Flexibility (operational conditions may change during use of the pump)

Generally, a pump with a relatively steep head-discharge curve is selected to promote stability during operation of the pump. If a pump has an H-Q curve where it is horizontal (or close to horizontal) over a given discharge range, this indicates that when pumping against this head the discharge could fluctuate significantly in an uncontrolled manner causing problems with surge and water hammer.

Consider the simplified pump system diagram shown in Figure 7 below.

![Figure 7](image_url)

One can state that: \[ H_T = H_S + h_{FS} + H_D + h_{FD} \]

- \( H_T \) – total head against which the pump must discharge
- \( H_S \) – static suction lift; which is the water level in the sump to the datum level of the pump
- \( h_{FS} \) – head loss due to frictional losses and minor losses in the suction pipe
- \( H_D \) – static delivery lift; which is the head required to pump water to the level of the discharge point
- \( h_{FD} \) – head loss due to frictional losses and minor losses in the delivery pipe

Academic year: 2006-2007
The total static head is the sum of $H_S$ and $H_D$.

Head losses due to frictional losses may be determined using Darcy’s equation

$$h_f = \frac{\lambda L v^2}{2gd}$$

where $\lambda$ is Darcy’s friction factor, $L$ is the length of the pipe, $v$ is the mean velocity of flow in the pipe and $d$ is the pipe diameter. Minor losses may include exit or entry losses.

The pump is normally located as close as possible to the sump or suction well to reduce the frictional loss in the suction pipe.

Once the system has been established for the pump, the **system curve** (or system characteristic) can be determined. The system curve is the relationship between head and discharge for the required system. The system curve is overlain on the pump characteristic to determine the operating point.

**Figure 8**

**Pumps in Series and in Parallel**

Pumps may be used in series or in parallel to achieve the desired Head-Discharge relationship. Consider two identical pumps in series, shown in Figure 9 below.

**Figure 9**
Since the discharge through pump 1 must equal the discharge through pump 2, one can write \( Q_1 = Q_2 = Q \). However, the head across points A and B is twice the head for a single pump, at a given discharge. Therefore, the H-Q characteristic curve for two identical pumps in series is obtained by doubling the head, at a given discharge, shown in Figure 10 below.

![Figure 10](image1.png)

Figure 10

Now, consider two identical pumps in parallel (at the same horizontal level), shown in Figure 11 below.

![Figure 11](image2.png)

Figure 11

Pumps 1 and 2 are identical, therefore at a given discharge \( Q \), the head delivered will be the same for each pump. Since the discharge entering at A \( (Q_T) \) must equal the discharge leaving at B \( (Q_T) \), then by continuity \( Q_T = Q_1 + Q_2 \), where \( Q_1 \) and \( Q_2 \) is the discharge through pump 1 and pump 2 respectively. For identical pumps \( Q_1 = Q_2 = Q \), and \( Q_T = 2Q \). However, the head across points A and B is equal to that for a single pump, at the given discharge, \( Q \). Therefore, the H-Q characteristic curve for two identical pumps in parallel is obtained by doubling the discharge, at a given head, shown in Figure 12 below.
Cavitation

Cavitation occurs when pressure falls below vapour pressure (at the appropriate temperature). The liquid boils and bubbles form in large numbers. At locations where the pressure increases again, these bubbles implode violently. This implosion results in very high velocities as the liquid rushes in to fill the void. High pressures and high temperatures are also generated at these locations.

This acting at or near solid surfaces can cause damage including fatigue failure. This phenomenon is accompanied by noise and vibration and is very destructive. Apart from physical damage, cavitation causes a reduction in the efficiency of the machine. Every effort must therefore be made to eliminate cavitation and this can be done by ensuring that the pressure is everywhere greater than the vapour pressure.

Conditions are favourable for cavitation where the velocity is high or the elevation is high and particularly where both conditions occur. For centrifugal pumps, the lowest pressure occurs near the centre of the impeller where the water enters (Figure 13).

Applying the energy equation between the surface of the liquid in the supply reservoir and the impeller (where the pressure is minimum), for steady conditions:

\[ p_0 + \frac{1}{2} \rho v_0^2 + \rho g z_0 = p_{min} + \frac{1}{2} \rho v^2 + \rho g (z_o + z) + \rho gh_{f3} \]

where \( v \) is the velocity of flow where the pressure is minimum.

However, the velocity in the supply reservoir, \( v_0 \), is equal to zero and the equation reduces to:

\[ p_0 = p_{min} + \frac{1}{2} \rho v^2 + \rho g z + \rho gh_{f3} \]
Dividing throughout by \( \rho g \), the equation has the form:
\[
P_o/\rho g = P_{\min}/\rho g + v^2/2g + z + h_{\text{fs}}
\]
and
\[
v^2/2g = \frac{P_o}{\rho g} - \frac{P_{\min}}{\rho g} - z - h_{\text{fs}}
\]

The pressure in the supply reservoir, \( P_o \), is usually atmospheric pressure, but not necessarily always.

However, \( v^2/2g \) for a given pump design, operating under specified conditions, may be taken as a particular portion of the head developed.

That is,
\[
v^2/2g \propto H
\]
\[
\implies v^2/2g = \sigma_c H
\]

This implies
\[
\sigma_c = \frac{\left( \frac{P_o}{\rho g} - \frac{P_{\min}}{\rho g} - z - h_{\text{fs}} \right)}{H}
\]

\( \sigma_c \) is related to the specific speed of the pump and is a property of the pump.

Now for cavitation not to occur, \( P_{\min} \), must be greater than \( P_v \) (where \( P_v \) is the vapour pressure of the liquid at the operating temperature).

To prevent cavitation \( \sigma > \sigma_c \) (or \( \sigma_c < \sigma \)) where
\[
\sigma = \frac{\left( \frac{P_0 - P_v}{\rho g} - z - h_{FS} \right)}{H}
\]

The above expression is known as Thoma's Cavitation Parameter.

The value \(\sigma H\) is known as the Net Positive Suction Head (NPSH), and is given by:

\[
\sigma H = \frac{P_0}{\rho g} - \frac{P_v}{\rho g} - z - h_{FS}
\]

Therefore,

\[
NPSH = \frac{P_0}{\rho g} - \frac{P_v}{\rho g} - z - h_{FS}
\]

To avoid cavitation one can ensure that the NPSH or \(\sigma\) is as large as possible. Techniques used to increase the NPSH include:

- Increase in the sump pressure (or the pressure of the supply reservoir, \(p_o\))
- Decrease in the vapour pressure, \(p_v\), by reducing the temperature of fluid
- Decrease the losses in the suction pipe, \(h_{FS}\)
- Decrease the static suction lift, \(z\)

At the point where cavitation will occur \(\sigma_c = \sigma\) and

\[
\sigma_c = \frac{\left( \frac{P_0 - P_v}{\rho g} - z - h_{FS} \right)}{H}
\]

\[
\sigma_c H = \frac{P_0}{\rho g} - \frac{P_v}{\rho g} - z - h_{FS}
\]

If \(p_o, p_v\) are considered to be constant, for a given application, and \(h_{FS}\) is considered to be negligible, one can write

\[
\sigma_c H = \frac{P_0}{\rho g} - \frac{P_v}{\rho g} - z
\]

Therefore, the maximum value of \(z\), \(z_{\text{max}}\), can be obtained from

\[
z_{\text{max}} = \frac{P_0 - P_v}{\rho g} - \sigma_c H
\]

Since for this given application, \(z_{\text{max}}\) gives the value of \(z\) at which cavitation would occur, if \(z < z_{\text{max}}\) for this given \(p_o\) and \(p_v\), then cavitation is prevented.
REFERENCES


